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Hybrid quarkonia with dynamical sea quarks

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We present a dynamical lattice calculation with two flavors for bottomonium states with an additional gluonic excitation. Using improved actions for the quarks and gauge fields at a lattice spacing of $a \approx 0.1$ fm, we find 10.977(61)(62) GeV for the energy of the lowest lying $b\bar{b}g$ hybrid, where the first error is statistical and the second denotes the systematic uncertainty due to the determination of scale. In a parallel quenched simulation we demonstrate explicitly that vacuum polarization effects are less than 10% of the splitting with the ground state.

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The intense experimental search for particles with exotic quantum numbers has triggered theoretical interest in hadrons which contain an excitation of the gluon flux [1]. The lattice approach to QCD has already resulted in consistent predictions for the masses of hybrid quarkonia from first principles [2–5]. The recent results suggest that the lowest lying hybrids are 400–500 MeV above the phenomenologically interesting $B\bar{B}$ threshold. However, for numerical simplicity, all those calculations were performed in the quenched approximation, where vacuum polarization effects have been ignored. In this paper we present our results from a full calculation in which dynamical sea quarks are included. Dynamical results for light hybrids were reported by the SESAM Collaboration [6]. Here our main goal is to establish whether a dynamical treatment of light sea quarks and heavy valence quarks will result in large shifts of the hybrid levels.

On the lattice, hybrid mesons are particularly difficult to treat as their excitation energies are much larger than for conventional states. This makes it very difficult to resolve the exponential decay of the meson propagator on coarse lattices. It was demonstrated that such problems can be circumvented on lattices with a finer resolution in the temporal direction [7], but the implementation of dynamical sea quarks on anisotropic lattices has yet to be achieved.

This work is part of a larger project to study sea quark effects in QCD on isotropic lattices [8]. For the gluon sector we chose a renormalization-group improved action, which is written in terms of standard plaquettes $\mathrm{Tr}\, P_{\mu\nu}$ and 1×2 rectangles $\mathrm{Tr}\, R_{\mu\nu}$:

$$S_{\rm g}=\frac{1}{g^2}\sum_{\mu\nu}\{c_0\,\mathrm{Tr}\,P_{\mu\nu}+c_1\,\mathrm{Tr}\,R_{\mu\nu}\}\quad (c_0+8c_1=1). \tag{1}$$

Our prescription $c_1=-0.331$ is motivated by a renormalization group (RG) analysis of the pure gauge theory [9]. For light sea quarks in the gauge field background we chose a lattice formulation which removes both the doublers and $\mathcal{O}(a)$ discretization errors:

$$S_{\rm q}=\bar{q}(\not\Delta+m_q)q+a\bar{q}\Delta^2 q-c_{sw}(a)a\frac{ig}{4}\bar{q}\sigma_{\mu\nu}F_{\mu\nu}q. \tag{2}$$

Further details and results for the conventional light hadron spectrum can be found in Ref. [8].

Here we study heavy hybrid states as they are of particular relevance for ongoing experiments at B-meson factories. To this end we implemented a nonrelativistic approach (NRQCD) for heavy b quarks on a fine lattice with $a \approx 0.10$ fm. This approach is well suited, owing to the small velocity of the quarks within the flat hybrid potential. Unlike calculations for the spin structure in quarkonia we expect only small corrections for spin-averaged quantities from higher order relativistic terms. This assumption has already been tested explicitly for charmonium [4]. To leading order we describe the forward evolution of the heavy quark as a diffusive problem [10]:

$$G(t+1)=\left(1-\frac{aH_0}{2n}\right)^n U_t^\dagger\left(1-\frac{aH_0}{2n}\right)^n G(t),$$

$$H_0=-\frac{\Delta^2}{2m_b},$$

$$\delta H=-c_7\frac{a\Delta^4}{16nm_b^2}+c_8\frac{a^2\Delta^{(4)}}{24m_b}. \tag{3}$$

The parameter n was introduced to stabilize the evolution against high-momentum modes. We chose $n=2$ throughout this analysis. As it is common practice, we have also included the correction terms c_7 and c_8 to render the evolution equation (3) accurate up to $\mathcal{O}(a^4)$, classically. Radiative corrections induce terms of $\mathcal{O}(\alpha a^2)$ and we applied a mean-field improvement technique to reduce such errors as first

TABLE I. Simulation parameters. For the dynamical run we measured $am_\pi/am_\rho=0.5735(48)$.

(β,κ)	(N_s,N_t)	a_σ [fm]	a_{1P-1S} [fm]	(am_Q,n)	u_{0L}	config.
(2.10, 0.1382)	(24,48)	0.11293(44)	0.10403(80)	(2.24,2)	0.854604	400
(2.528,quenched)	(24,48)	0.11319(97)	0.0883(16)	(1.92,2) (2.24,2)	0.876700	192

suggested by Lepage *et al.* [11]. We decided to divide all gauge links by the average link in the Landau gauge: $u_0=\langle 0|(1/3)U_\mu|0\rangle_{\rm L}$, which is a suitable choice to reduce the unphysical tadpole contributions in lattice perturbation theory.

Nonrelativistic meson operators can be constructed using the standard gauge-invariant definitions of Ref. [4]. For the magnetic hybrid this amounts to an insertion of the color-magnetic field B_i into the bilinear of two-spinors:

$$^1H(x)=\sum_{i=1}^{3}\psi^\dagger(x)B_i\chi(x). \tag{4}$$

In the following we will denote as $1B$ and $2B$ the ground state and the first excitation onto which this lattice operator can project. We also employ fuzzed link variables and several different smearings for the quark fields. This results in different projections of the source operators onto the ground state and it is useful when extracting higher excitation with the same quantum number.

For our study we calculated the bottomonium spectrum on 400 dynamical configurations at $(\beta,\kappa)=(2.10,0.1382)$ on a $24^3\times 48$ lattice. This corresponds to the lightest sea quark mass of our full data set at $\beta=2.1$ and we measured $m_\pi/m_\rho=0.5735(48)$ [8]. For the comparative quenched analysis we accumulated 192 independent configurations at $\beta=2.528$. This coupling was chosen so as to match the lattice spacing of the dynamical run. In both cases we find $a\approx 0.11$ fm from the string tension $\sqrt{\sigma}=440$ MeV. Here we take the $1P-1S$ splitting to set the scale. As expected, such a definition results in slightly different lattice spacings for our two data sets.

Finite size effects are known to be small for heavy quarkonia and they have been explicitly checked for $1S$, $1P$ and $1B$ in charmonium [4]. Since here we study the bottomonium system on even larger lattices ($L\approx 2.5$ fm), we do not expect any volume dependence for the ground state masses in our analysis. The simulation parameters for our two data sets are collected in Table I.

The results for hadron masses are obtained from correlated multiexponential fits to different smearings α and timeslices t:

$$C(\alpha,t)=\langle M_\alpha^\dagger(t)M_\alpha(0)\rangle=\sum_{i=2}^{n_{\rm fit}}a_i(\alpha)e^{-m_it}. \tag{5}$$

In Table II we present the results from multiexponential fits ($n_{\rm fit}\leq 4$).

As is standard in lattice calculations with heavy quarkonia, we have tuned the bare quark mass in Eq. (3), so as to reproduce the experimental value of $M_{\rm kin}/(1P-1S)=21.5$, where $aM_{\rm kin}$ is determined for the nonrelativistic dispersion relation of the S state on the lattice. In Fig. 1 we demonstrate the mass independence of the spin-independent ground states at ($\beta=2.528$, $N_f=0$), along with their higher excitations. This independence is important, as it allows us to extract predictions at a slightly nonphysical point. It also means that our result is stable against possible radiative corrections to the quark mass. Since in NRQCD calculations all energies are measured with respect to the ground state $1S$, we introduced the ratio $R_X=(X-1S)/(1P-1S)$ to quote the normalized splitting with respect to $1P-1S$.

Here we focus on the ratio $R_{1B}=(1B-1S)/(1P-1S)$ which determines the mass of the lowest lying hybrid. The velocity expansion of NRQCD works very well for the hybrid states, owing to the slow quarks in a flat hybrid potential. Therefore we do not expect significant changes for the mass of the spin-averaged hybrid state due to higher order

TABLE II. Normalized splittings with respect to $1P-1S$. The experimental values are taken from n^3S_1 and the spin-averaged nP. The dynamical results come from our lightest sea quark mass ($\kappa=0.1382$).

(β,N_f) am_Q	(2.528, 0) 1.92	(2.528, 0) 2.24	(2.10, 2) 2.24	experiment
$aM_{\rm kin}$	4.008(16)	4.628(17)	4.642(18)	
1P-1S	0.2057(34)	0.2007(37)	0.2318(16)	439.37(13) MeV
$M_{\rm kin}$ [GeV]	8.74(15)	10.35(19)	8.805(76)	9.46030(26)
(2S-1S)/(1P-1S)	1.461(51)	1.459(48)	1.389(22)	1.2802(16)
(3S-1S)/(1P-1S)	3.12(27)	3.15(23)	3.255(98)	2.0353(24)
(2P-1S)/(1P-1S)	2.28(18)	2.21(19)	2.336(60)	1.8186(20)
(3P-1S)/(1P-1S)	4.36(40)	4.47(42)	4.94(17)	–
(1B-1S)/(1P-1S)	3.43(45)	3.63(39)	3.59(14)	–
(2B-1S)/(1P-1S)	5.8(1.1)	6.1(1.1)	6.83(52)	–

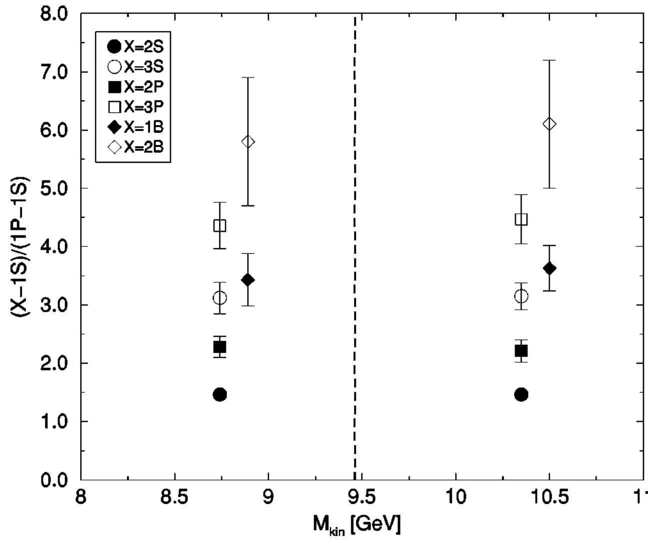

FIG. 1. Mass independence of the spin-averaged bottomonium spectrum. Here we plot the excitations against the kinetic mass of $^3S_1$ from two different quark masses. The measured energies are 8.74(15) GeV and 10.35(19) GeV. The hybrid results are shifted for clarity and the experimental upsilon mass is shown as a vertical dashed line.

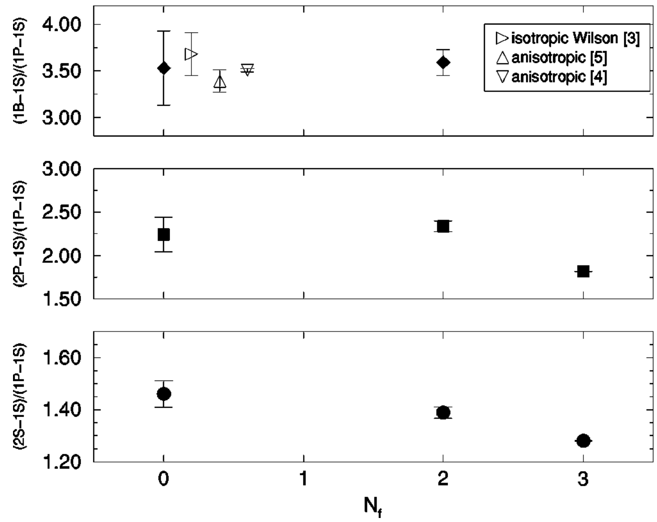

FIG. 2. Dependence of R_X on numbers of dynamical flavors, N_f. The experimental values are plotted at $N_f=3$. For R_{1B} we also show quenched results from other groups as triangles (offset for clarity).

relativistic corrections. To leading order in the NRQCD Hamiltonian, the singlet state of Eq. (4) is degenerate with the corresponding spin-triplet states $(0^{-+},1^{-+},2^{-+})$. These are all states with zero orbital angular momentum, and they include the exotic combination 1^{-+}, due to the coupling of the spin to the gluon angular momentum. A near degeneracy was also reported for hybrid states with additional orbital angular momentum [3]. Such states give rise to more exotic states, such as 2^{+-}. The above-mentioned degeneracies will be lifted once higher order relativistic corrections are reintroduced into the NRQCD Hamiltonian [12,13].

The main uncertainty in lattice calculations of hybrid excitations so far is the absence of dynamical sea quarks. Previous estimates of quenching errors frequently referred to the uncertainty in the determination of the lattice spacing as a limiting factor of quenched simulations. However, it is not clear *a priori* whether the high gluon content of the hybrid itself would cause large shifts in its mass once dynamical sea quarks are introduced. Comparing the results in Table II, we find, perhaps surprisingly, that this is not the case. Indeed, our quenched estimate for $R_{1B}=3.43(45)$ is in excellent agreement with the dynamical simulation, $R_{1B}=3.59(14)$. This means that quenching errors are smaller than our statistical errors of about 10% for this quantity. This is a pleasing feature for lattice calculations and it is also confirmed by another dynamical calculation [14] which could not resolve any change of the static hybrid potential. In this picture one can also understand why relativistic corrections and discretization errors are so small—in the flat potential the quarks move very slowly and are widely separated, hence they are not very sensitive to the details of the lattice cutoff. This is in contrast to the *S*-state splittings, where dynamical effects are to be expected because of their sensitivity to the physics at short distance scales. Here we observe a 5% shift of R_{2S}, in line with an earlier calculation at a similar lattice spacing [15]. It is important to realize that this accounts for only part of the dynamical effects in the real world. As has been argued before [16], our sea quark mass is sufficiently small for the purpose of bottomonium calculations, but it is apparent that large gluon momenta can also excite strange quarks from the vacuum—a full description ought to include three dynamical flavors.

For a short-range quantity like R_{2S} it is equally important to study scaling violations. Our previous work [17] showed that discretization errors may also account for a fraction of the remaining $\leqslant 10\%$ deviation from experiment. We leave a more precise determination of those two different sources of systematic errors to future studies. For R_{1B} we have convincing evidence from previous simulations [4,5] that at $a \approx 0.1$ fm scaling violations are negligible for spatially large hybrid states. In the following we take the deviation of R_{2S} from its experimental value as a conservative estimate for residual systematic errors in our two-flavor simulation.

In Fig. 2 we plot our results against N_f, along with previous quenched estimates from different lattice spacings and a variety of isotropic or anisotropic lattice actions. The stability of all these results and the good agreement with our value for two-flavor full QCD provides support to the arguments above. Converting our dynamical result into dimensionful units we quote 10.977(61)(62) GeV for the lowest lying hybrid, where the first error is statistical and the second denotes the systematic uncertainty in the determination of the lattice spacing.

Taking the conservative average of all points in Fig. 2, we find 11.02(18) GeV as the lattice prediction for the mass of the lowest spin-averaged bottomonium hybrid. We want to stress the importance of this result, as it shows that even after the introduction of two dynamical flavors the lowest lying hybrid will be above the $B\bar{B}$ threshold (10.56 GeV), where a number of experiments are currently running. In fact our prediction is intriguingly close to the $B\overline{B}^{**}$ threshold (≈ 11.0 GeV), below which hybrid states are thought to be very narrow.

Our results show that the error from the quenched approximation is small within the statistical error of isotropic lattice calculations ($\approx 1\%$ of the bound state energy). To study systematic errors for hybrid states at an even higher level of accuracy is possible on anisotropic lattices, but it remains to be seen how dynamical quarks can be implemented in such a framework.

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